

Quarkonia in anisotropic hot QCD medium in a quasi-particle model

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The fate of heavy quarkonia states has been investigated in QCD at high temperature when the plasma has a small momentum space anisotropy within a quasi-particle model. A real time static potential has been obtained from a Hard thermal loop expression for the gluon self-energy, by employing the quasi-parton equilibrium distribution functions extracted from hot QCD equations of state of $O(g^5)$ and $O(g^6 \ln(1/g))$. Employing the potential, quarkonia melting has been studied. It is found that the equations of state and the momentum anisotropy cause significant modifications in the screening properties of the plasma.

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I. INTRODUCTION

The dissociation of heavy quarkonia states due to the presence of screening of static color fields in hot QCD medium has long been proposed as a signature of a deconfined medium, and QGP formation[1]. Since then, this has been a area of active research[2] and undergone several refinements[3, 4, 6–9]. However, a precise understanding of this phenomenon is still a subject of debate. There are disagreements between various approaches due to several ambiguities[10]. Attempts are being made on to fix them[4]. A precise definition of the dissociation temperature is still elusive and is a matter of intense theoretical and phenomenological investigations –from the perspective of lattice spectral function studies[8, 11–14] or potential inspired models[10, 15–17] or effective field theories[18]. Experimentally, quarkonia suppression has been seen in RHIC experiments at BNL[5] and hopefully the understanding will be refined at LHC heavy ion program in CERN in the near future.

The heavy quark-antiquarks, such as $c\bar{c}$ are bound together by almost static gluons[19–21]. The fate of $q\bar{q}$ bound states, in a QGP medium, can be explored by determining the behavior of gluon self energy at high temperature. Recall that the real part leads to Debye screening on which the traditional understanding of quarkonia melting in QGP medium is based. The imaginary part of the potential implies that thermal effects generate a finite width for the quarkonium peak in the dilepton production rate. They[21] further showed that the physics related to the finite width originates from the Landau-damping of low-frequency gauge fields, and could be studied non-perturbatively by making use of the

classical approximation. This leads to a new proposal to explain the quarkonia melting in QGP medium[21, 22]. According to which the quarkonia bound states dissociate effectively at lower temperatures as compared to that obtained from Debye screening picture. In this picture, the quarkonium state will disappear whenever the thermal width exceeds the binding energy. The temperature lies in the range $g^2 M < T < gM$, where M is the heavy quark pole mass[19]. The estimate has been refined employing the understanding from the effective theories in [23–25]. These studies suggest that the quarkonia dissociation in the thermal medium is not dominated by the Debye screening rather by the above mentioned mechanism. Further, modifications to these estimates by considering the anisotropic momentum distribution for hard partonic degrees of freedom has been reported in Ref.[19].

At this juncture, we note that the estimates of [19] for the dissociation temperature in the above analyses is based on the assumption that the QGP medium is fully equilibrated and that it can be treated as an ideal gas of quarks and gluons. On the other hand, one knows that the latter assumption is particularly far from being true. We are also in possession of equations of state for finite temperature QCD which have been evaluated up to $O(g^5)$ [26, 27] and $O(g^6 \ln(1/g))$ [28]. It is, therefore, natural that we should revisit the problem where the strongly interacting nature of the quarks and gluons in their deconfined state is explicitly and fully incorporated in determining this important quantity. Taking into consideration that the QGP which is produced in heavy ion collisions does not possess the full isotropy, but only a cylindrical symmetry at most, it is also not out of place to study screening phenomenon in an anisotropic QGP medium. This sets the motivation for our investigations.

In the present paper, we shall consider an anisotropic QGP (non-equilibrium environment) and employ a recently proposed quasi-particle model for hot QCD equations of state[29, 30].

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We argue that all the assumptions taken in Ref.[19] will be equally valid in the present case too. Here, the anisotropy is in the momentum distribution functions of the hard partonic degrees of freedom same as in Ref.[19]. This situation may be expected in the initial stages of the collisions due to the Bjorken expansion and extensively studied. There is a wealth of literature present[31–36] where the impact of the anisotropy in various observables for QGP has been well studied. In most of these studies, the isotropic distribution function have been considered as the combination of the ideal Bose/Fermi distributions[37]. The assumption that the hard degrees possess anisotropic momentum distribution functions, is only justifiable at the late stages of the collision, so that soft degrees of freedom have already get time to thermalize. As correctly stated in Ref.[19], this does not lead to a self-consistent non-equilibrium ground state and the system has techyonic modes. Nevertheless, as emphasized in Ref.[19], these issues will be absent in the small anisotropy limit. Therefore, in view of this, we only restricts in the small anisotropy limit.

Here, we shall consider the more realistic distribution functions which are extracted from the improved perturbative QCD equations of state[29, 30]. In this manner, we include the interactions in the various observables for QGP in the presence of anisotropy. More precisely, we address questions such as, (i) the form of the heavy quark potential with the realistic equations of state, and (ii) the dependence of the dissociation temperature of quarkonia states on the anisotropy and interactions. We shall see that the interactions non-trivially modify the static heavy quark potential and the dissociation temperatures.

The paper is organized as follows. In section II, we shall review a quasi-particle description of hot QCD EOS which will be employed in the subsequent sections to study the fate of quarkonia in an anisotropic QGP. In section III, we shall discuss the heavy quark potential and melting of heavy quarkonia states. In section IV, we shall conclude the present work.

II. QUASI-PARTICLE DESCRIPTION OF HOT QCD

In this section, we briefly review a quasi-particle description of hot QCD equations of state proposed in Refs.[29, 30]. In these studies, a quasi-particle model has been developed to capture all the interaction effects present in hot QCD equations of state in terms of non-interacting quasi-gluons and quasi-quarks having temperature dependent effective fugacities. The model came out to be quite successful to yield the hot QCD equations of state accurately. The model has further been employed to study the chromo-electric response functions[30] and transport properties of QGP in RHIC[38, 39]. In this context, pure $SU(3)$ lattice EOS has also been studied[39]. A very strong prediction of the model is regarding the transport properties of QGP. In particular, the model has been

employed to study the shear viscosity, η of QGP[38, 39]. We obtained η by employing the model in an appropriate transport equation[38, 40]. We showed that both η and its ratio with entropy density, η/s are very sensitive to the interactions and they could be thought of as good diagnostics to distinguish various EOS at RHIC and LHC. A very small value for the ratio η/s was obtained. We found that η/s may violate the universal bound conjectured from AdS/CFT studies[41].

Here, we consider the quasi-particle description[29, 30] of $O(g^5)$ hot QCD[26, 27] and $O(g^6 \ln(1/g))$ hot QCD EOS [28]. We denote former EOS as EOS1 and latter as EOS2. EOS1 and EOS2 qualitatively matches with the more realistic lattice EOS for $T \geq 2 T_c$. In fact, EOS2 shows better agreement as compared to EOS1[30]. Note that EOS1 and EOS2 have been evaluated at vanishing baryon density. We start with the equilibrium distribution functions extracted from these two equations of state. They possess a very simple form,

$$\begin{aligned} f_0^g &= \frac{z_g \exp(-\beta p)}{\left(1 - z_g \exp(-\beta p)\right)} \\ f_0^q &= \frac{z_q \exp(-\beta p)}{\left(1 + z_q \exp(-\beta p)\right)}, \end{aligned} \quad (1)$$

where g stands for quasi-gluons, and q stands for quasi-quarks. z_g is the quasi-gluon effective fugacity and z_q is quasi-quark effective fugacity. These distribution functions are isotropic in nature. At this juncture, we want to emphasize the physical meaning of z_g and z_q . These fugacities should not be confused with any conservation law (number conservation) and have merely been introduced to encode all the interaction effects at high temperature QCD. At this juncture, we wish to emphasize that the effective fugacities with the similar motivations to address complicated interactions in BEC (Bose Einstein Condensate)[42] and interacting Fermi systems[43] have already been introduced in condensed matter physics.

Their physical meaning could be understood in terms of modified quasi-parton dispersion relations,

$$\begin{aligned} \omega_p^g &= p + T^2 \partial_T \ln(z_g) \\ \omega_p^q &= p + T^2 \partial_T \ln(z_q), \end{aligned} \quad (2)$$

where ω_p denotes the single quasi-parton energy. It is clear from Eq. (2) that $z_{g/q}$ non-trivially contribute to the single particle energy. This tells us that the model is in the spirit of Landau theory of Fermi liquids. Further, $z_{g/q}$ leads to the quasi-parton number densities N_g and N_q as follows,

$$N_{g/q} = \int \frac{d^3 p}{8\pi^3} f_0^{g/q} = \frac{\pm \nu_{g/q}}{\pi^2} \text{PolyLog}[3, \pm z_{g/q}], \quad (3)$$

where the function $\text{PolyLog}[3, z] = \sum_{l=1}^{\infty} \frac{z^l}{l^3}$. Both z_g and z_q have a very complicated temperature dependence

and asymptotically reach to the ideal value unity[30]. Here, $\nu_g \equiv 2(N_c^2 - 1)$ and $\nu_q \equiv 4N_c N_f$, are the number of gluonic and quark-antiquark degrees of freedom respectively for $SU(N_c)$. The quasi-particle description does not change the group velocity of quasi-partons, $\vec{v}_p^{g/q} = \partial_{\vec{p}} \omega_p^{g/q} \equiv \hat{p}$ (this observation may play an important role in setting up the kinetic equation to determine the transport parameters for QGP with the quasi-particle model). This property allows us to distinguish the present model from other models based on an effective mass description[44, 45].

This description, which enables us to adapt realistic QCD EOS to make predictions for RHIC, leads to very interesting results as far as the bulk and transport properties of QGP in RHIC concern. Encouraged by that we now seek to employ the understanding to study the fate of quarkonia in a anisotropic QGP medium.

III. HEAVY QUARKONIA IN ANISOTROPIC QGP

It is phenomenologically quite interesting to study QGP system which is slightly away from equilibrium. It is interesting to inspect quarkonia in a hot QCD environment where the hard parton distribution functions posses anisotropy in their momentum distribution function. This phenomenon is inspected in the recent past in Ref.[19] and very interesting physical understanding has been obtained. Further, this problem has also been considered for an ideal EOS in Ref.[20]. These results deviate slightly from [19] because of the fact that the physics setting is different. They[20] also assumed non-thermal distributions for the soft glouns. For similarities between these two analyses, we refer the reader to [19].

Here, we shall primarily follow the work of [19] and extend in the case of EOS1 and EOS2 by combining the analysis with the quasi-particle description discussed in the previous section. The main motivation here is to adapt the realistic hot QCD EOS and investigate the fate of quarkonia bound states in this interacting environment.

A. The real time static potential

Here, we combine the quasi-particle description of hot QCD described in the previous section to the formalism of heavy quark-potential in anisotropic QGP[19]. The aim here is to see how the inclusion of interactions coming from the realistic equations of state for QGP affect the heavy quark-potential and quarkonia melting in an anisotropic QGP.

The starting point to obtain the real time static quark-antiquark potential is the Hard thermal loop gluon self energy, $\Pi^{\mu,\nu}$ for a general momentum distribution function[37, 46], $f(\vec{p})$,

$$\Pi_R^{\mu\nu}(K) = g^2 \int \frac{d^3\vec{p}}{(2\pi)^3} v^\mu \partial^\alpha f(\vec{p}) \left(\delta^\nu_\alpha - \frac{v^\nu k_\alpha}{v^\sigma k_\sigma + i0^+} \right) \quad (4)$$

where $v \equiv (1, \vec{p}/p)$, and R indicates that the self-energy appears in the inverse of the retarded propagator. The hard mode momentum distribution has been assumed to be the following form[47],

$$f(p) = f_0(\sqrt{p^2 + \xi(\vec{p} \cdot \hat{n})^2}), \quad (5)$$

where ξ a real parameter known as the anisotropy parameter and \hat{n} is the direction of anisotropy. Without the loss of generality, \hat{n} can be chosen to be along the z-direction. The quantity $\vec{p} \cdot \hat{n} \equiv p_z$, where p_z is the momentum component in the beam direction in RHIC. The distribution functions $f(p)$ can be constructed from an arbitrary isotropic distribution function, f_0 by the rescaling of only one direction in momentum space. We note that ξ is in $[-1, \infty]$, and $\xi = 0$ corresponds to the isotropic case. The values corresponding to the Bjorken expansion induced anisotropy are $\xi > 0$.

Now, making use of the change of variable $\bar{p} \equiv \sqrt{p^2 + \xi p_z^2}$, and denoting $p_z \equiv p \cos \theta_p$, and following the analysis in Ref.[19], we get the following form of the component Π_R^{00} (which is needed for the real time static potential) near the static limit assuming the anisotropy, ξ to be small (first order in ξ)[19],

$$\Pi_R^{00}(K) \approx m_D^2 \left\{ -1 + \xi \left[\frac{1}{6} - \frac{1}{2} \cos(2\theta_k) \right] - \frac{i\pi}{2} \frac{k_0}{k} \left[1 + \xi \cos(2\theta_k) \right] \right\}, \quad (6)$$

where θ_k is the angle between \vec{k} and \vec{z} . The quantity m_D is defined in terms of the equilibrium (isotropic) distribution function as,

$$m_D^2 \equiv -g^2 \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{df_{iso}(\vec{p})}{d\vec{p}}. \quad (7)$$

So far the analysis is quite general and allow any suitable arbitrary form for the f_{iso} . Note that often, f_{iso} is taken to be a combination of ideal Bose-Einstein and Fermi-Dirac distribution functions as[37],

$$f_{iso} = 2N_c n_b(p) + 2N_f (n_q(p) + n_{\bar{q}}(p)), \quad (8)$$

where $n_b(p) = \frac{1}{\exp(-\beta p) - 1}$, and in the zero baryon chemical potential $n_q = n_{\bar{q}} = \frac{1}{\exp(-\beta p) + 1}$. This combination leads to the leading order HTL expression ($m_D^2 = g^2(T)T^2(N_c/3 + N_f/6)$) for the Debye mass in hot QCD. Here, N_c denotes the number of colors and N_f the number of flavors.

A more appropriate choice for f_{iso} would be a suitable combination of quasi-parton distribution functions[30] (f_0^g and f_0^q displayed in Eq. (1)). These are mandated

by the fact that QGP is not an ideal gas of gluons and quarks rather a strongly interacting system[48–53].

We make the following substitutions for f_{iso} ,

$$f_{iso} = \begin{cases} 2N_c f_0^g & \text{for pure gauge,} \\ 2N_c f_0^g + 2N_f f_0^q & \text{for full QCD} \end{cases} \quad (9)$$

Note that the expression for the Debye mass in Eq.(7) with f_{iso} in Eq.(8) is same as leading order HTL expression obtained from effective kinetic models [54–57] for hot QCD medium with ideal EOS. These results are equally applicable in the present case too due the fact the the quasi-partons in our quasi-particle model are non-interacting. The only consistency requirement is that in the asymptotic limit the results should match with those for the ideal EOS. This could be achieved with the chosen expressions for f_{iso} in Eq.(9). We shall discuss the Debye mass with these choices of f_{iso} in detail in the next subsection.

We now discuss the real and imaginary part of the static potential which depends upon the form of m_D and the anisotropy parameter. In the leading log order the real and imaginary parts have been analyzed in Ref.[19], and they obtain the following form,

$$\begin{aligned} \text{Re}[V >] &\sim -\frac{g^2 C_F}{4\pi r} [1 + O(\hat{r})] |_{r \sim \frac{1}{g^2 M}} \\ \text{Im}[V >] &\sim -g^2 C_F T \frac{m_D^2 r^2}{3} \ln\left(\frac{1}{m_D r}\right) [1 + O(\frac{1}{\ln(\hat{r})})] \\ &\times (1 - \frac{\xi}{3}) \end{aligned} \quad (10)$$

where M is the pole mass of heavy quarks[19], and $\hat{r} = m_D r$. Here, only the s-wave ground states have considered. In this case the term proportional to $(1 - 3\cos^2\theta_k)$ in Eq.(6) average out to zero[19] at first order in perturbation theory and corrections will be of the order ξ^2 . Similar mathematical forms for the real and imaginary part of the static potential are obtained for the present case too. The interactions appear via m_D and merely renormalizes the effective charges of the quasi-partons, which we shall see next.

1. The Debye mass m_D

Before, we employ the quasi-parton distribution functions in Eq.(1) to obtain the temperature dependence of m_D , We wish to discuss a very important point here. There is an issue of defining the parameter $\beta \equiv 1/T'$ which appears in f_{iso} in terms of the equilibrium temperature T [19]. The parameter T' will be different from the temperature T in the equilibrium. This is due to the fact that, in the presence of anisotropy the system is away from equilibrium. If we assume the system to be very close to the equilibrium (case of very small anisotropy) then the parameter T' could be taken to be T . As discussed in [19], the parameter T' can also be fixed from

isentropic condition. This is quite tedious here due to the non-trivial temperature dependence of the effective fugacities, z_g and z_q . We do not consider this particular case here and stick to the near equilibrium condition.

Now, employing f_0^g and f_0^q in Eq.(7) we obtain, m_D in the purely gluonic case and full QCD as,

$$\begin{aligned} m_D^2 &= g^2 T^2 \left(\frac{N_c}{3} \times \frac{6 \text{PolyLog}[2, z_g]}{\pi^2} \right) \text{ for pure gauge} \\ m_D^2 &= g^2 T^2 \left\{ \left(\frac{N_c}{3} \times \frac{6 \text{PolyLog}[2, z_g]}{\pi^2} \right) \right. \\ &\quad \left. + \left(\frac{N_f}{6} \times \frac{-12 \text{PolyLog}[2, -z_q]}{\pi^2} \right) \right\} \text{ for full QCD.} \end{aligned} \quad (11)$$

Here, $g(T)$ is the QCD running coupling constant, $N_c = 3$ ($SU(3)$) and N_f is the number of flavor, the function $\text{PolyLog}[2, z]$ having form, $\text{PolyLog}[2, z] = \sum_{k=1}^{\infty} \frac{z^k}{k^2}$. These are the same expression which we have obtained from the chromo-electric response functions[30] for the interacting QGP. The extra factors appearing due to the inclusion of interactions can be attributed to the charge renormalization in QGP medium[30]. Define the effective charges as,

$$\begin{aligned} Q_g^2 &= g^2 \frac{6 \text{PolyLog}[2, z_g]}{\pi^2} \\ Q_q^2 &= g^2 \frac{-12 \text{PolyLog}[2, -z_q]}{\pi^2}. \end{aligned} \quad (12)$$

The expressions for the Debye mass can be recasted as,

$$m_D^2 = \begin{cases} Q_g^2 T^2 \frac{N_c}{3} & \text{for pure gauge,} \\ T^2 \left(\frac{N_c}{3} Q_g^2 + \frac{N_f}{6} Q_q^2 \right) & \text{for full QCD} \end{cases} \quad (13)$$

Both Q_g^2 and Q_q^2 acquire the ideal value g^2 only asymptotically. Moreover, $\{Q_g^2, Q_q^2\} \leq g^2(T)$. In other words, these expressions for the Debye masses reach their ideal values (leading order HTL expression) only asymptotically). To see this, we plot the ratio $\mu_d \equiv m_D/m_D^I$,

$$\mu_d = \begin{cases} \frac{Q_g}{g} & \text{for pure gauge,} \\ \frac{(\frac{Q_g}{g} + \frac{N_f}{6} \frac{Q_q}{g})}{(1 + \frac{N_f}{6})} & \text{for full QCD} \end{cases} \quad (14)$$

as a function of temperature employing the temperature dependence of z_g and z_q from Ref.([30]) in Eq.(13) for pure gauge theory and full QCD respectively in Fig. 1.

The quantity μ_d scales with T/T_c coming from the scaling properties of z_g and z_q [30]. The ratio, μ_d is relevant in investigating the dissociation temperatures for quarkonia states, which we shall see next.

B. Quarkonia dissociation

Here, we choose the following criteria for the quarkonia dissociation. The temperature at which the magnitude

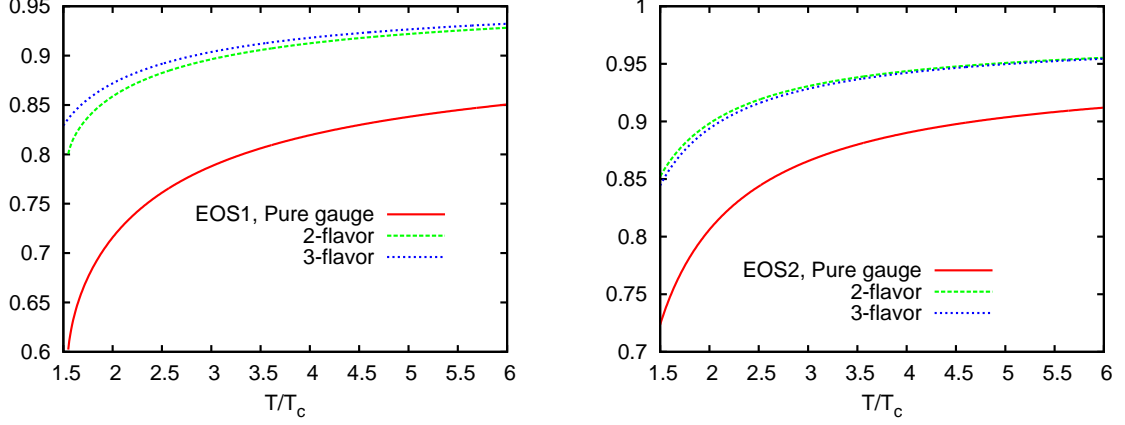


FIG. 1: Behavior of μ_d as function of T/T_c . The left panel shows the results for EOS1 and right panel for EOS2. In both the cases, μ_d reaches the ideal value unity only asymptotically, and $\mu_d \leq 1$.

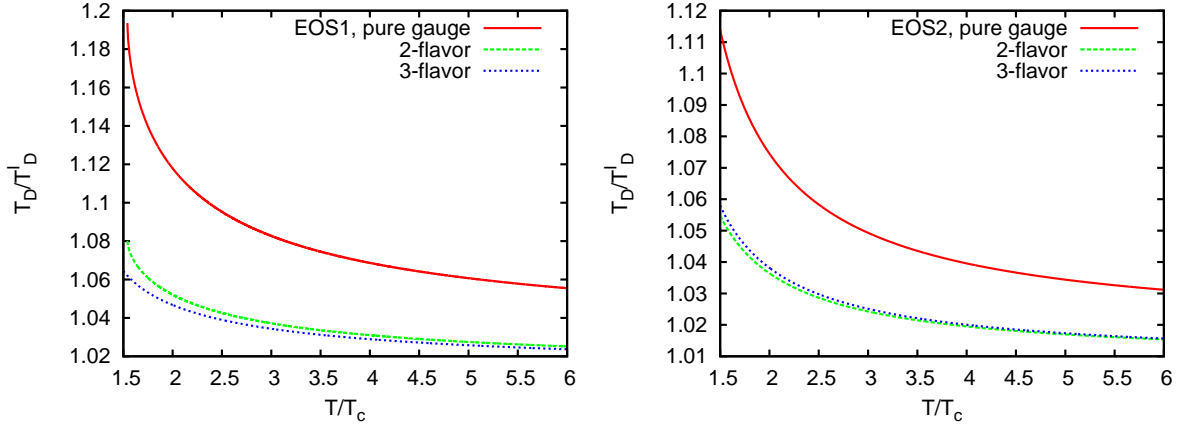


FIG. 2: Behavior of T_D/T_D^I as function of T/T_c . The left panel shows the results for EOS1 and right panel for EOS2. In both the cases, it reaches the ideal value unity only asymptotically, and always ≥ 1 .

of the real part of the static potential (real part gives the binding energy) displayed in Eq.(10), equals the magnitude of the imaginary part of the potential (imaginary part gives the width of the state) is identified as the dissociation temperature.

Employing Eq.(10), We obtain the following condition,

$$\frac{g^2}{r} \sim g^2 \frac{T m_D^2 r^2}{3} \ln\left(\frac{1}{m_D r}\right) \times \left(1 - \frac{\xi}{3}\right). \quad (15)$$

Where $r \sim 1/g^2 M$ [19]. Using $T' = T$ and Debye masses given in Eq.(11), we obtain the following expression for

the melting temperature in pure gauge theory,

$$T_D = \frac{g^{4/3} M}{3^{1/3} \left(\frac{Q_g}{g}\right)^{1/3}} \times \left(\frac{1}{2} \ln\left(\frac{g}{Q_g}\right) + \ln\left(\frac{1}{g}\right)\right)^{-\frac{1}{3}} \times \left(1 + \frac{\xi}{9}\right). \quad (16)$$

On the other hand in full QCD,

$$T_D = \frac{g^{4/3} M}{3^{1/3} \left(\frac{Q_g}{g} + \frac{N_f Q_q}{6g}\right)^{1/3}} \left(\frac{1}{2} \ln\left(\frac{1}{\left(\frac{Q_g}{g} + \frac{N_f Q_q}{6g}\right)}\right) + \ln\left(\frac{1}{g}\right)\right)^{-\frac{1}{3}} \times \left(1 + \frac{\xi}{9}\right). \quad (17)$$

The corresponding ideal expression for pure gauge theory reads,

$$T_D^I = \frac{g^{4/3}M}{3^{1/3}} \times \left(\ln\left(\frac{1}{g}\right) \right)^{-\frac{1}{3}} \times \left(1 + \frac{\xi}{9}\right). \quad (18)$$

On the other hand in full QCD,

$$T_D^I = \frac{g^{4/3}M}{3^{1/3}(1+N_f/6)^{\frac{1}{3}}} \times \left(\frac{1}{2} \ln\left(\frac{1}{(1+N_f/6)}\right) + \ln\left(\frac{1}{g}\right) \right)^{-\frac{1}{3}} \times \left(1 + \frac{\xi}{9}\right). \quad (19)$$

Note that Eqs.(16),(17) possess same ξ dependence as in[19]. There the role played by ξ in modifying the potential as well as the quarkonia melting temperatures will be exactly the same. The interesting observations are regarding the inclusion of the realistic EOS (EOS1 and EOS2). As it is clear from Eqs.(16) and (17) that interactions non-trivially modify the s-wave ground states quarkonia melting temperature. To see the effects of interactions more closely and quantitatively, we consider the ratio T_D/T_D^I in the case of both pure gauge theory and full QCD. It is straight forward to see that in the weak coupling limit ($g \ll 1$), the ratio can be given as,

$$\frac{T_D}{T_D^I} = \begin{cases} \left(\frac{Q_g}{g}\right)^{-1/3} & \text{for pure gauge,} \\ \left(\frac{(\frac{Q_g}{g} + \frac{N_f}{6} \frac{Q_g}{g})}{1 + \frac{N_f}{6}}\right)^{-1/3} & \text{for full QCD} \end{cases} \quad (20)$$

Note that the ratio $T_D/T_D^I \equiv \mu_d^{-\frac{1}{3}}$. Clearly from the Fig.1, the ratio is always ≥ 1 . It reaches to unity only asymptotically. Therefore, the dissociation temperatures with non-ideal EOS will always be higher as compared to those for their ideal counter parts. This implies that the presence of interactions decreases the effect of Debye screening in hot QCD. We have plotted the ratio, T_D/T_D^I in Fig. 2. Quantitatively, dissociation temperatures are $\sim 20\%-6\%$ higher than the ideal value near $T \sim 1.5T_c$ for EOS1. On the other hand, for EOS2, these are $\sim 12\%-5\%$ higher than the ideal values near $T \sim 1.5T_c$ (see Fig. 2). In the case of 2- and 3- flavor QCD, the dissociation temperatures are more closer to their ideal counter parts as compared to the pure gauge theory. This is true in the case of both EOS1 and EOS2. In other words, differences with the ideal EOS are more pronounced in the case of pure gauge theory.

In the anisotropic case with non-ideal EOS, the dissociation temperatures vary from the corresponding isotropic case by a factor $\frac{\xi}{9}$. Therefore, in the case of non-ideal EOS (EOS1 and EOS2) for $\xi \sim 1$ there is an increase of $\sim 10\%$ in the dissociation temperatures as compared to the equilibrium case. These conclusions regarding the

anisotropy are same as that for an ideal EOS in[19]. This is not unexpected because the quasi-parton distribution functions employed in this work only differ from the ideal Bose/Fermi distributions by purely temperature dependent suppression factors (effective fugacities, z_g, z_q) otherwise they are non-interacting.

IV. CONCLUSIONS

In conclusion, we have studied the form of the static inter-quark potential and quarkonia dissociation in an anisotropic QGP in the small anisotropy limit employing quasi-parton equilibrium distribution functions obtained from $O(g^5)$ and $O(g^6 \ln(1/g))$ hot QCD equations of state. We find that the interactions non-trivially modify the real time static potential for a heavy quark-antiquark pair. In particular, in the leading order the real part of the potential remains intact as for ideal EOS. In contrast, imaginary part gets non-trivial modifications. Interestingly, the dissociation temperatures are also non-trivially modified in the presence of interactions. We studied both pure gauge theory and full QCD sectors. We show that the interactions coming from the realistic QGP EOS merely renormalize the partonic charges. They enter in the potential as well as in the dissociation temperatures through the Debye mass. The above conclusions are found to be valid for both the equations of states. These results are valid for small anisotropies. For larger anisotropies, numerical simulations are required. It is clear from our study that for the non-equilibrium states at small momentum space anisotropy one can estimate the relative change caused by the anisotropy at leading-logarithmic order employing the realistic QGP EOS.

It would be of interest to extend the present study to the case of non-vanishing baryon density and recently proposed realistic lattice equations of state with physical quarks masses[51, 53], and also the chromo-electric response functions[30]. Finally, it would be of interest to consider the Bjorken hydrodynamic expansion and study the viscous corrections to the response functions. These investigations will be done separately in near future. A more interesting study would be to develop a two-component model which can handle anisotropies in both hard and soft partonic modes and their impact on the inter-quark forces in hot QCD medium.

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